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Hierarchical Risk-Parity Portfolio Structure

Quantitative Investments

September 2022

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Risk-Parity Portfolio

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Portfolio Theory

What are the key metrics?

Sharpe Ratio

- The Sharpe ratio divides a portfolio's excess returns by a measure of its volatility to assess risk-adjusted performance
- Excess returns are those above an industry benchmark or the risk-free rate of return, such as the 10-year treasury bill
- The calculation may be based on historical returns or forecasts
- A higher Sharpe ratio is better when comparing similar portfolios
- The Sharpe ratio has inherent weaknesses and may be overstated for some investment strategies

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where:

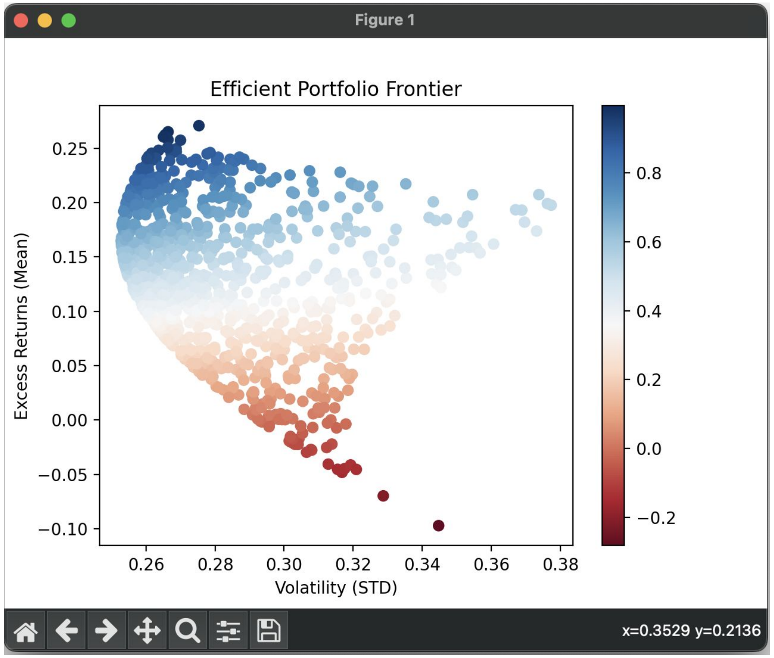
R_p = return of portfolio

R_f = risk-free rate

σ_p = standard deviation of the portfolio's excess return

Excess Returns vs Volatility

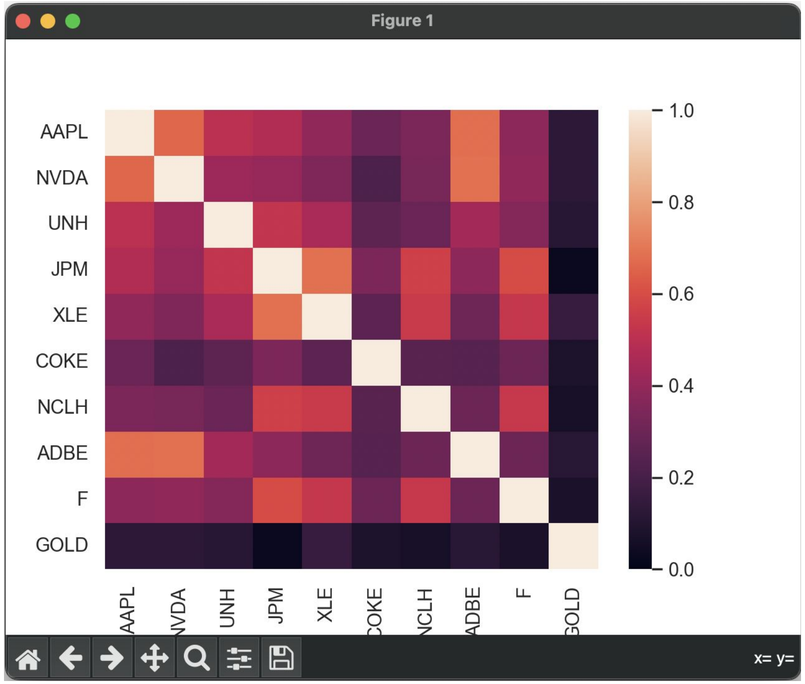
- Scatterplot of Asset Returns – Risk Free ROR and the asset's Volatility
- Currently being used to determine asset clusters based on similar Returns vs Risk



Risk-Parity Portfolio

What is it?

- Allocate Risk instead of Capital
- Allocate the risk exposure to different asset and/or classes
- Better Sharpe Ratio than a standard equal allocation strategy



Shortcomings

- Assigns risk measure compared to the entire portfolio instead of similar asset classes

Hierarchical Risk-Parity (HRP) Portfolio

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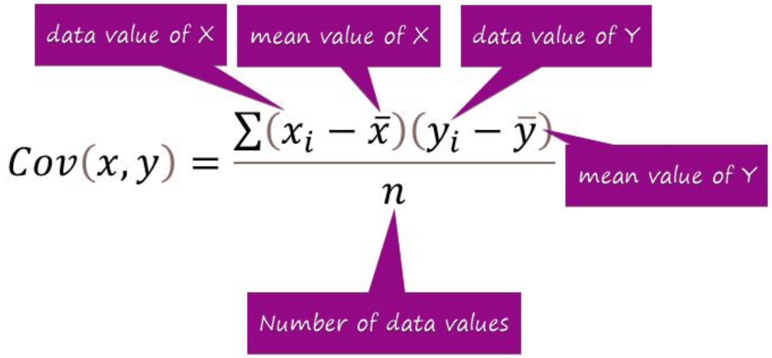
Covariance

Theory – Hierarchical Risk Parity Model

Covariance

- A measure of the joint variability of two random variables
- The magnitude of the covariance is not easy to interpret because it is not normalized and hence depends on the magnitudes of the variables
- The variance is a special case of the covariance in which the two variables are identical



$$Cov(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

$$\sigma^2 = \frac{\sum (xi - \bar{x})^2}{N}$$

Theory – Hierarchical Risk Parity Model

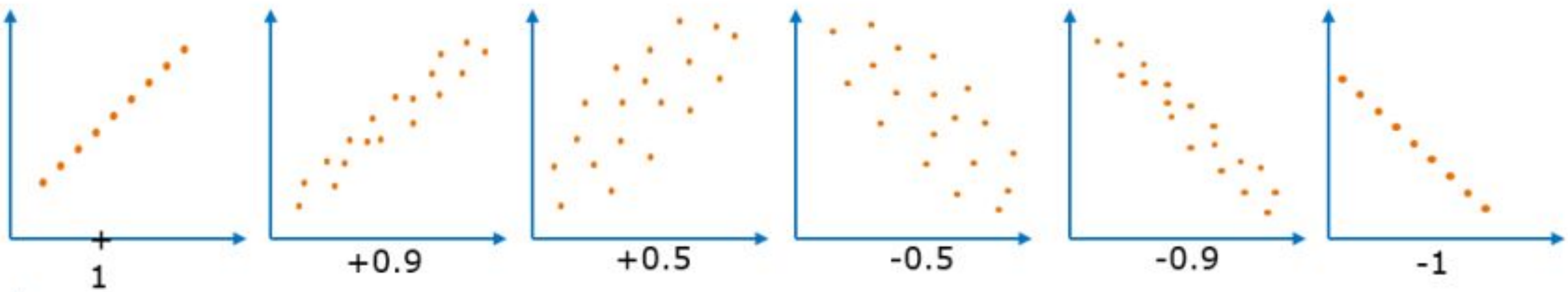
Correlation coefficient r is number between -1 to +1 and tells us how well a regression line fits the data and defined by

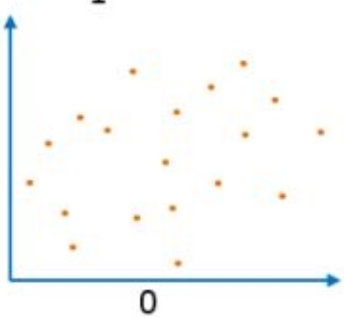
$$r_{xy} = \frac{s_{xy}}{s_x s_y}$$

where,

- s_{xy} is the covariance between x and y
- s_x and s_y are the standard deviations of x and y respectively.

Perfect +ve Correlation | High +ve Correlation | Low +ve Correlation | Low -ve Correlation | High -ve Correlation | Perfect -ve Correlation





1. Correlation coefficient r=0 mean there is no linear relationship between x and y however other functional relationship may exist.
2. One point to note here is if there is no relationship at all between x and y then r will certainly be 0 but not vice versa (refer point 1)

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Theory – Hierarchical Risk Parity Model

Covariance Matrix

Covariance matrix is a square matrix giving the covariance between each pair of elements of a given random vector.

The matrix is symmetric as cov(x,y) = cov(y,x) and positive definite (all elements are greater than equal to 0)

The diagonal elements are the variances of the individual random variables in the random vector as cov(x,x) = Var(x)

$$\begin{array}{c|cc} & x & y \\ \hline x & var(x) & cov(x,y) \\ y & cov(x,y) & var(y) \end{array}$$

$$\begin{array}{c|ccc} & x & y & z \\ \hline x & var(x) & cov(x,y) & cov(x,z) \\ y & cov(x,y) & var(y) & cov(y,z) \\ z & cov(x,z) & cov(y,z) & var(z) \end{array}$$

Covariance Versus Correlation

Correlation is a "normalized covariance" in the sense that this operation transforms covariance to [−1,1]

Covariance

- Indicates direction of linear relationship
- Positive covariance indicates an increase in one variable indicates an increase in the other
- Covariance can be between -infinity to infinity

Correlation

- Indicates direction and strength of linear relationship
- Correlation coefficient can be between -1 and 1
- Positive correlation coefficient closest to 1 indicates a strong positive correlation and value close to -1 indicates a strong negative correlation

	0	1	2	3	4	5	6	7	
0	0.00477	0.00264	0.00259	0.00305	0.00190	0.00184	0.00253	0.00286	0.
1	0.00264	0.00671	0.00302	0.00400	0.00275	0.00323	0.00263	0.00270	0.
2	0.00259	0.00302	0.00610	0.00373	0.00275	0.00218	0.00219	0.00285	0.
3	0.00305	0.00400	0.00373	0.00735	0.00363	0.00311	0.00312	0.00358	0.
4	0.00190	0.00275	0.00275	0.00363	0.00711	0.00226	0.00302	0.00322	0.
5	0.00184	0.00323	0.00218	0.00311	0.00226	0.00657	0.00268	0.00299	0.
6	0.00253	0.00263	0.00219	0.00312	0.00302	0.00268	0.00525	0.00318	0.
7	0.00286	0.00270	0.00285	0.00358	0.00322	0.00299	0.00318	0.00606	0.

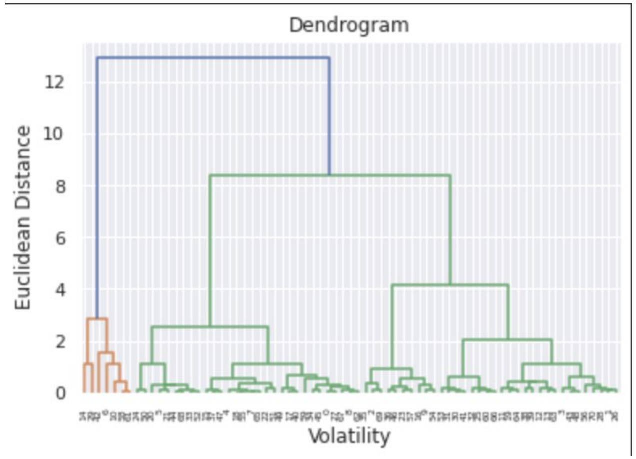
	AAPL	NVDA	UNH	JPM	XLE	COKE	NCLH	ADBE	
AAPL	1.00000	0.66221	0.39226	0.27791	0.18986	0.29212	0.25874	0.65348	0.
NVDA	0.66221	1.00000	0.32725	0.23170	0.16938	0.21663	0.26517	0.69663	0.
UNH	0.39226	0.32725	1.00000	0.36262	0.28560	0.23542	0.18522	0.30464	0.
JPM	0.27791	0.23170	0.36262	1.00000	0.59675	0.38183	0.56954	0.16646	0.
XLE	0.18986	0.16938	0.28560	0.59675	1.00000	0.25437	0.47279	0.09612	0.
COKE	0.29212	0.21663	0.23542	0.38183	0.25437	1.00000	0.23778	0.22760	0.
NCLH	0.25874	0.26517	0.18522	0.56954	0.47279	0.23778	1.00000	0.22236	0.
ADBE	0.65348	0.69663	0.30464	0.16646	0.09612	0.22760	0.22236	1.00000	0.
F	0.31544	0.33969	0.25758	0.57478	0.47539	0.33905	0.52759	0.24812	1.
GOLD	0.17289	0.19087	0.12683	0.00789	0.13187	0.11898	-0.03904	0.16339	0.

Implementation – Hierarchical Risk Parity Model

Clustering

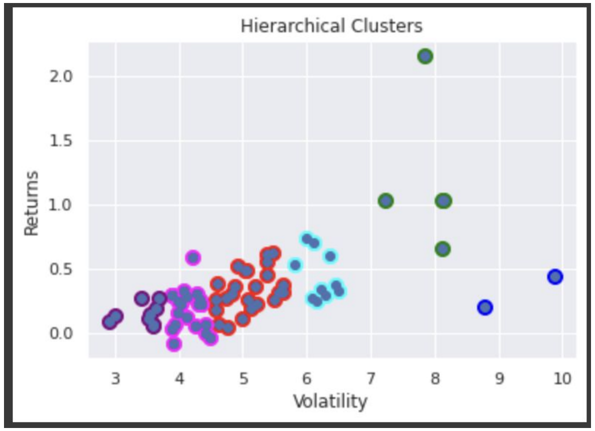
Agglomerative Clustering

- Plot asset's Excess Returns and Volatility on the scatterplot
- Treat each object as single cluster
- Merge two nearest clusters based on Euclidean distance
- Recursively repeat until one large cluster is formed



Resources

- Number of clusters: human input (trial and error)
- Clustering: sklearn
- Dendrogram: scipy



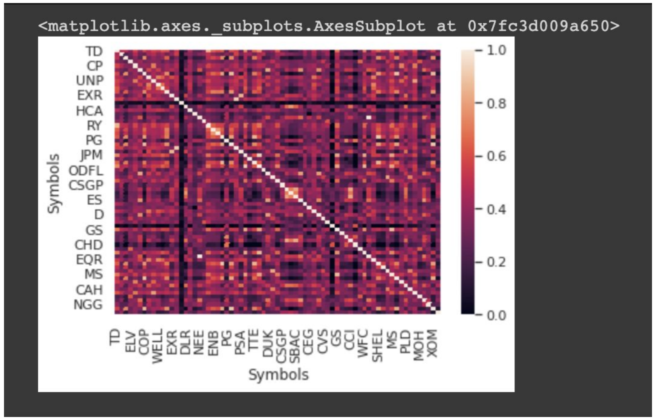
Implementation – Hierarchical Risk Parity Model

Seriation

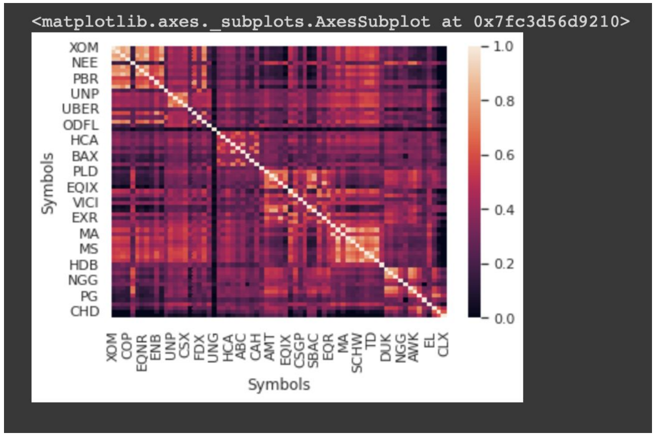
What is Seriation?

- Grouping of similar assets on a covariance heatmap
  - Based on the clustering in previous step
- Puts largest correlation along the diagonal variance line
- Necessary to assign weights to *similar* asset classes

Unordered Heatmap



Ordered Heatmap



Implementation – Hierarchical Risk Parity Model

Recursive Assignment of Weights

What is Seriation?

- Recursively bisects already-sorted list of tickers into 2 subclusters
- Calculate new covariance matrix of subclusters using *Formula 1*
- Using new covariance matrix, calculate new weighting factor (*Formula 2*)
- For each subcluster, repeat process

Formula 1

$$V_{adj} = w^T V w$$

where,

$$w = \frac{diag[V]^{-1}}{trace(diag[V]^{-1})}$$

Formula 2

$$\alpha_1 = 1 - \frac{V_1}{V_1 + V_2}; \alpha_2 = 1 - \alpha_1$$

Result

	ordered_tickers	y_hc
0	JNJ	0
1	ABBV	0
2	MRK	0
3	MSFT	1
4	CSCO	1
5	UNH	1
6	PFE	1
7	TMO	1
8	V	1
9	ABT	1
10	F	2
11	COOP	2
12	NVDA	2
13	AVGO	3
14	AAPL	3
15	MS	3
16	LLY	3
17	MA	3
18	CVBF	3
19	JPM	3
20	XLE	4
21	ADBE	4
22	BAC	4
23	GOLD	4
24	AXP	4
25	FITB	5
26	COKE	5

ABBV	0.048985
MRK	0.032403
JNJ	0.032403
V	0.049128
ABT	0.032270
MSFT	0.032270
TMO	0.047700
UNH	0.034055
CSCO	0.034055
PFE	0.045173
F	0.045173
COOP	0.039965
NVDA	0.039965
JPM	0.046532
LLY	0.037643
AVGO	0.037643
MA	0.027145
CVBF	0.027145
AAPL	0.026529
MS	0.026529
ADBE	0.052017
GOLD	0.051773
BAC	0.051773
AXP	0.023374
XLE	0.023374
FITB	0.027489
COKE	0.027489

	cluster	weight_each_cluster
0	Cluster 1	0.113791
1	Cluster 2	0.113667
2	Cluster 2	0.115811
3	Cluster 3	0.170275
4	Cluster 4	0.121818
5	Cluster 4	0.107348
6	Cluster 5	0.155563
7	Cluster 6	0.101726

Back-testing – Why the Hierarchical Risk Parity Model?

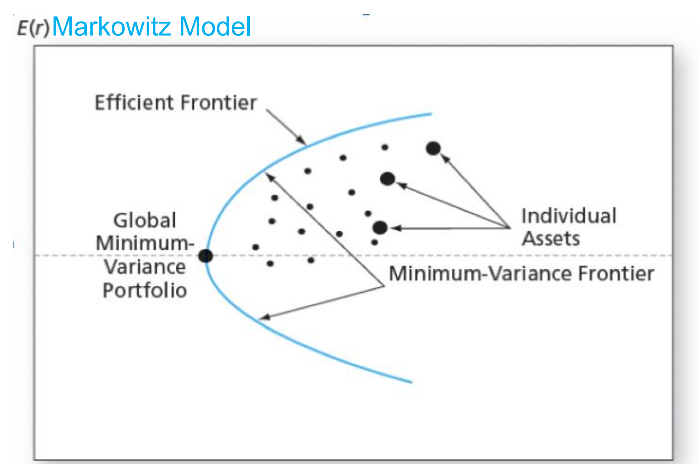
The Problems Faced by Alternative Traditional Allocation Approaches

Modern Portfolio Theory (MVO)

- Developed by Harry Markowitz: Markowitz Model
- The expected return of a portfolio is a weighted average of the expected returns of each of the securities in the portfolio
  - $E(R_p) = S X_i R_i$ where X_i is the weight of allocation and R_i is the return
- For a two-asset portfolio, risk is the square root of the sum of the weighted (X^2i) times the variances (s^2) of each security and the correlation (ρ) between each pair of securities

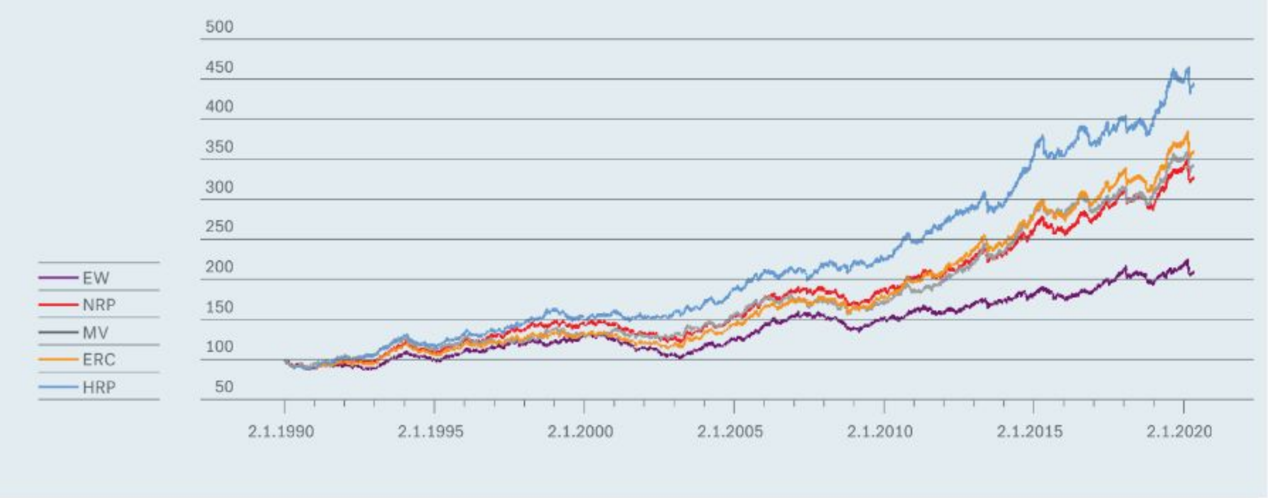
$$\sigma_p^2 = w_A^2 \times \sigma_A^2 + w_B^2 \times \sigma_B^2 + 2 \times w_A \times w_B \times \sigma_A \times \sigma_B \times \rho_{AB}$$

  - where $r_{i,j}$ is the correlation between the two assets
  - Where ρ_{AB} is the correlation between the two assets
  - Low correlation means lower portfolio volatility
- σ^2 explains variance, σ explains volatility
- Downsides: MPT is too sensitive to the stationarity of return time series so that even small forecasting and estimation errors without structural breaks in market behavior can lead to dramatically different efficient frontiers



Back-testing – Why the Hierarchical Risk Parity Model?

- **Comparison of Historical Index Performance**



- **Key performance metrics from different allocation methods**

Performance Metric

	EW	NRP	MV	ERC	HRP
Compound annual growth rate	2.5%	4.0%	4.1%	4.3%	5.0%
Volatility	4.9%	4.9%	5.1%	5.0%	5.0%
Sharpe ratio[4]	0.51	0.81	0.80	0.86	1.00
Maximum drawdown	-23.5%	-17.9%	-13.5%	-15.6%	-12.2%
Sortino ratio[5]	0.81	1.30	1.31	1.39	1.64
Calmar ratio[6]	0.11	0.23	0.31	0.28	0.42
Mean leverage	70%	106%	142%	120%	128%

Investment portfolio

Constituent	Asset class	Currency	Constituent	Asset class	Currency
Australia 10Y Govt Bonds	Fixed Income	AUD	STOXX Europe 600	Equities	EUR
Canada 10Y Govt Bonds	Fixed Income	CAD	FTSE 100	Equities	GBP
France 10Y Govt Bonds	Fixed Income	EUR	Hang Seng	Equities	HKD
Germany 10Y Govt Bonds	Fixed Income	EUR	NASDAQ-100	Equities	USD
Italy 10Y Govt Bonds	Fixed Income	EUR	Russell 2000	Equities	USD
UK 10Y Govt Bonds	Fixed Income	GBP	S&P 500	Equities	USD
USA 10Y Govt Bonds	Fixed Income	USD	S&P/TSX 60	Equities	CAD
Gold	Commodities	USD	SMI	Equities	CHF
Silver	Commodities	USD	SPI 200	Equities	AUD
DAX	Equities	EUR	Topix	Equities	JPY

Back-testing – Why the Hierarchical Risk Parity Model?

Great Bond Massacre 1994

1 January 1994– 31 January 1995	EW	NRP	MV	ERC	HRP
Commodities	-1.1%	-1.1%	-2.6%	-2.2%	-1.6%
Equities	-4.6%	-3.1%	0.6%	-2.8%	-1.4%
Fixed Income	-2.8%	-5.2%	-8.2%	-5.4%	-7.3%
Total	**-8.6%**	**-9.4%**	**-10.3%**	**-10.5%**	**-10.3%**

The Global Financial Crisis 2007-2009

9 October 2007– 9 March 2009	EW	NRP	MV	ERC	HRP
Commodities	0.2%	0.2%	0.1%	0.2%	0.3%
Equities	-14.5%	-16.0%	-12.4%	-14.2%	-6.8%
Fixed Income	1.4%	5.5%	8.0%	8.0%	13.1%
Total	**-12.9%**	**-10.3%**	**-4.3%**	**-6.1%**	**6.5%**

Coronavirus Pandemic 2020

1 January 2020– 30 April 2020	EW	NRP	MV	ERC	HRP
Commodities	-1.9%	-2.1%	-1.3%	-1.9%	-0.2%
Equities	-2.6%	-2.2%	-6.4%	-2.3%	-4.7%
Fixed Income	0.5%	1.2%	5.7%	1.6%	4.2%
Total	**-4.0%**	**-3.0%**	**-2.0%**	**-2.6%**	**-0.8%**

Short Comings - Hierarchical Risk Parity Model

Asset Selection

- Selected via largest market cap per sector (15 samples per sector)
- Alternatively, could be selected as a function of EV/EBITDA to find the fair market value of assets adjusted for revenues

Selected Metrics

- Using Excess Returns vs Volatility to identify similarly-correlated assets
- Could use better volatility metrics, like Beta of asset compared to the market, making it a stronger measure of risk

Next Steps & Future Goals

1. Improve Clustering

- Refine the distance quantification (function that calculates distance)
  - Metrics quantifying how one asset informs another
    - Predictive/Granger causality, partial, lead-lag correlation
  - Non-linear distance calculations
- Investigate other clustering algorithms (function that uses distance to define correlation)

2. Apply HRP for Risk Management

- Tail dependency, kurtosis, systematic risk, networks, and causality

3. Apply HRP for Statistical Industry Classifications

- Statistical industry classifications are more sophisticated than traditional
- Apply weights based on industry rather than individual stocks
- Agglomerative; bottom-up; top-down clustering

Questions or Suggestions?

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