Equity Derivatives Gamma Exposure Model

Fundamental Investments

CALGORY CAPITAL

November 2023

Table of Contents

Understanding Gamma Exposure

Allows us to track dealer and total gamma exposure in all listed $SPX options by visualizing the data by strike

Gamma as a Second Order Derivative

What is Gamma?

Gamma is a second order derivative that represents the rate of change in an option's delta per one point move in the underlying asset

SPX Options comprise roughly 20% of the SPX market cap

Understanding Gamma exposure is critical to dealers as they hedge the delta risk from the billions of dollars of notional value from client orderflow, each day

Market Maker Delta Hedging

Two Ways to Delta Hedge an Options Portfolio

Offset Delta Risk by Buying/Selling Underlying Asset

For example, assume you are long calls with a net delta position of 0.5. The options trader would need to sell 50 shares of the underlying asset to become *delta neutral*

Offset Delta Risk by Buying/Selling Futures Contract

Provides a much cheaper alternative to hedging while minimizing the market impact. This remains the primary method for market makers

OPEX and Triple Witching Events

Each month, dealers undergo a unanimous procedure known as OPEX which is marked by the month's expiring options. Dealers need to decide how to alter these positions and their respective hedges. As market makers must achieve a risk-neutral options portfolio, this is historically an incredibly choppy market event

Triple Witching occurs once each quarter, overlapping with the month's OPEX. However, triple witching includes equity options, index options as well as futures contracts. Dealers must decide whether to roll over or liquidate their E-Mini contracts in addition to the options they hold.

Understanding where the Gamma exposure is highest tells us where dealers will hedge

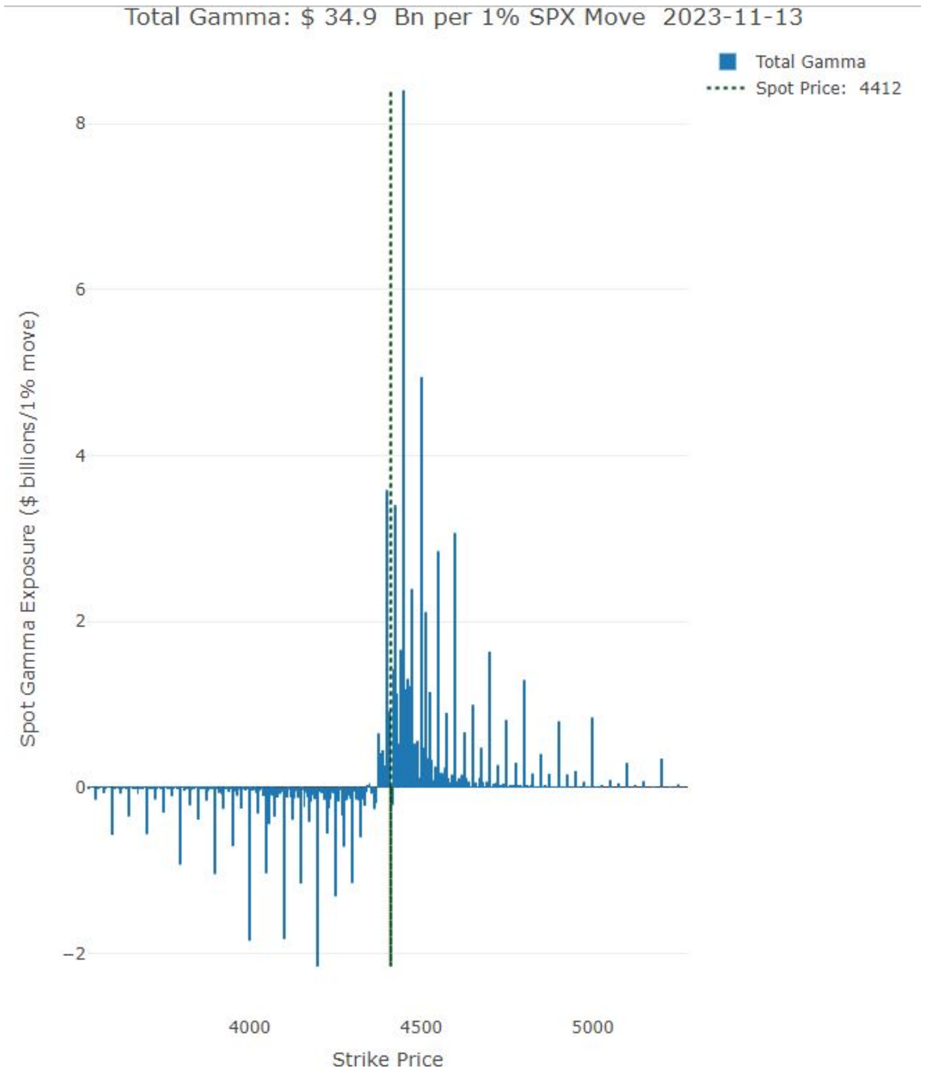
Dealer Derivatives Feedback Loop

Summarizing Sell-Side Options Market Making & Hedging

Sell-Side firms are issuing options worth billions in notional val each day. To maintain a balanced book, they hedge any residu exposure with shares or futures. As the underlying moves, so c the delta-hedging requirement and thus are forced to trade the underlying to achieve delta-neutrality.

The loop occurs where the type and amount of options that inv trade influences the underlying price action. **If dealers are prir selling calls, they are going into the market to buy the underlying or the respective amount of futures contracts.**

Gamma Exposure Model - Total Exposure per 1% Ch in Price



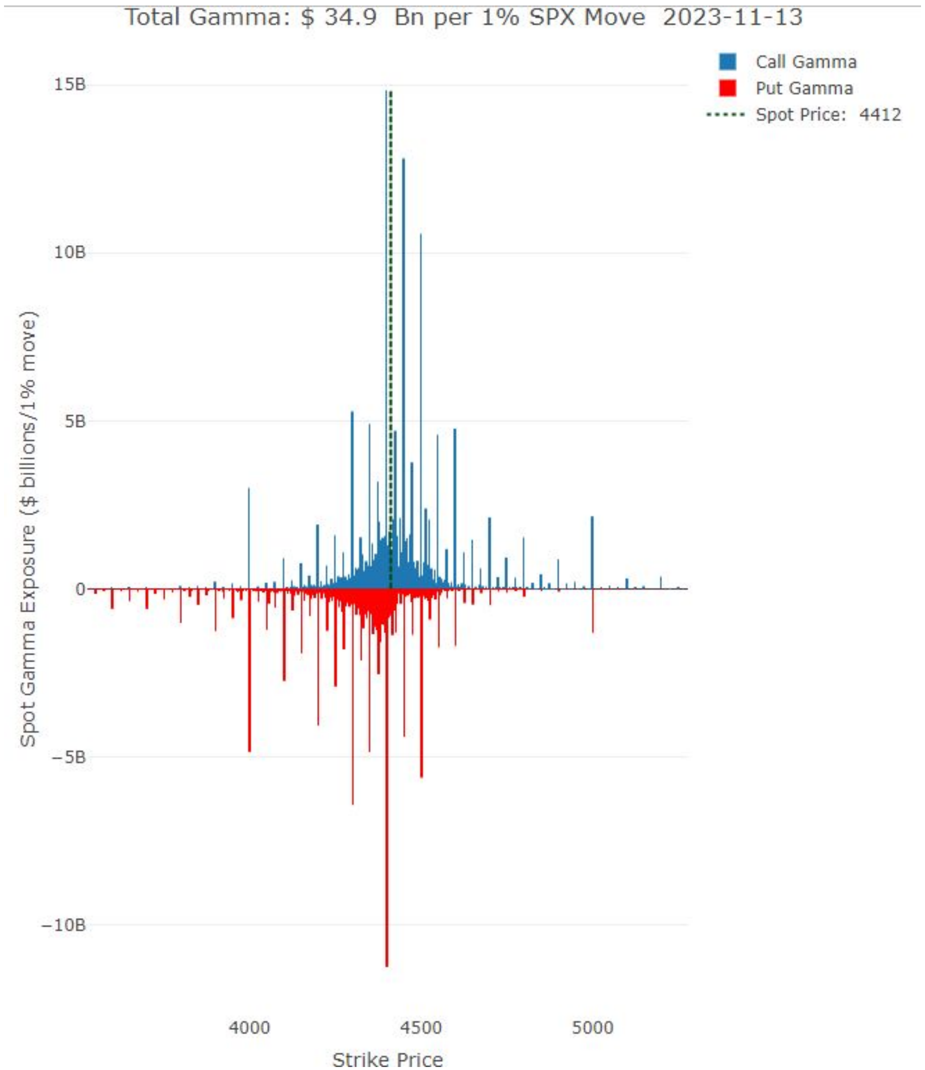
Visualizing Market Gamma Exposure

- Graph shows total gamma in Bn per 1% of move
- Visualizes how much gamma is outstanding at each strike
- Each bar represents how much delta needs to be bought or sold if the market moves 1% from spot
- Chart shows *call walls* and *put walls*
- *Ex. 4450 is a call wall*
  - Levels where there is large call or put gamma

Understanding Call and Put Walls

- Establishes rudimentary probabilistic trading range
- Assumption suggests that MM is long calls and short underlying
  - As underlying increases, delta needs to be hedged and MM sells
  - Results in price hovering around the call wall as dealers hedge, suppressing volatility
- Put holders generally open OTM, when puts hit the money they roll down the contracts in mass
  - This is because put reward/risk curve is not as steep ATM/ITM
  - Immediate effect of closing these positions ATM prompts dealer buying
  - Causes lower imp vol → shrink deltas → more MM buying

Gamma Exposure Model - Total Exposure by Calls and Puts



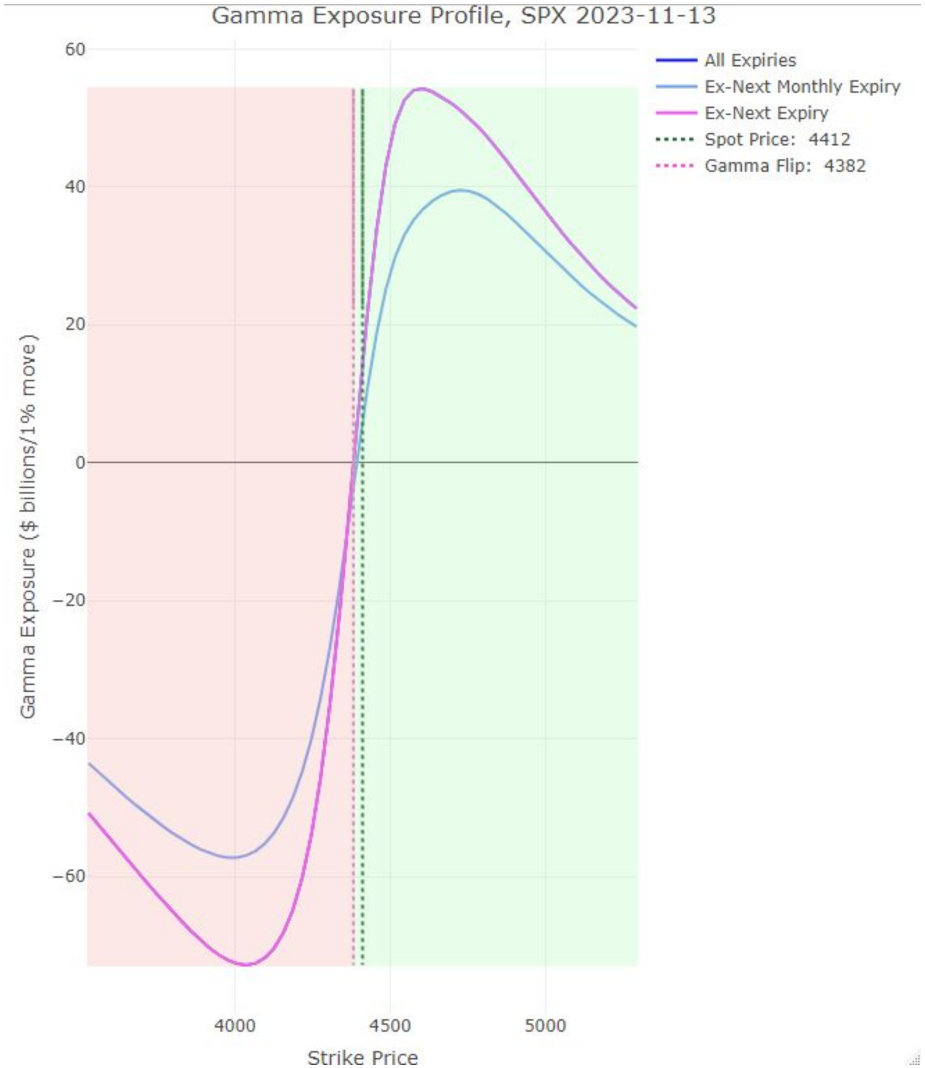
Visualizing Market Gamma By Calls & Puts

- Graph shows gamma exposure by strike in terms of calls and puts
- Reminder → Long call/Short put produces positive gamma while Short call/Long put produces negative gamma
- In addition to all things from the previous slide being true, this tells us:
  - Executed SM positions by strike
  - Market directionality by positioning in calls & puts

Institutional Order Flow

- Most institutions are buying point to point call spreads at 98/102 of strike
- Model visualizes where positions are being executed
- Can be loosely interpreted to understand the liquidity distribution in calls and puts
  - **If we can visualize where people are trading, and what positions they are trading, we can position ourselves accordingly to follow the liquidity**

Gamma Exposure Model - Gamma Exposure Profile & Flip Point



Gamma Exposure Profile Explained

- Estimate of gamma over a range of spot prices
- Gives an approximation of the amount of dealer hedging flows across spot levels
  - Short gamma to the left
  - Long gamma to the right
- Gamma flip point is the zero gamma point
  - Indicates where dealer gamma exposure flips positive or negative
- Seeks to model how gamma changes when the price of the underlying changes

Model Specifications and Flexibility

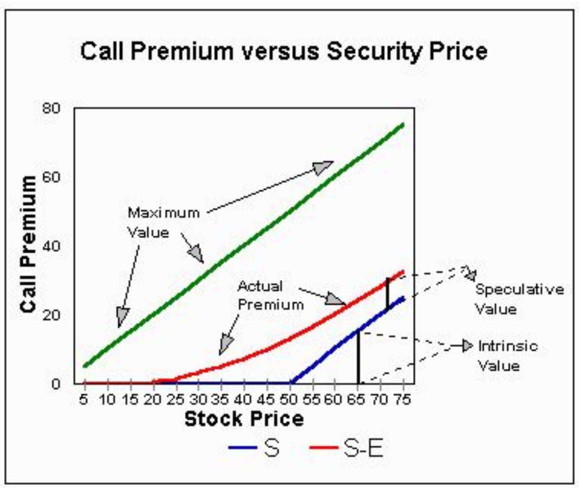
- Currently the model is only considering gamma sample size for 80/120 of spot. This can be widened or narrowed in the code, easily.
- Gamma Exposure Profile is calculated off the 60 most relevant strikes (30 above spot 30 below spot) and can be widened or narrowed easily
- Model calculates these based off of CBOE options data
  - Can be applied to **any single stock, index, or ETF** with a liquid options chain
- Serves as synergistic model to assist all divisions in executing trades and understanding exposure and risk levels

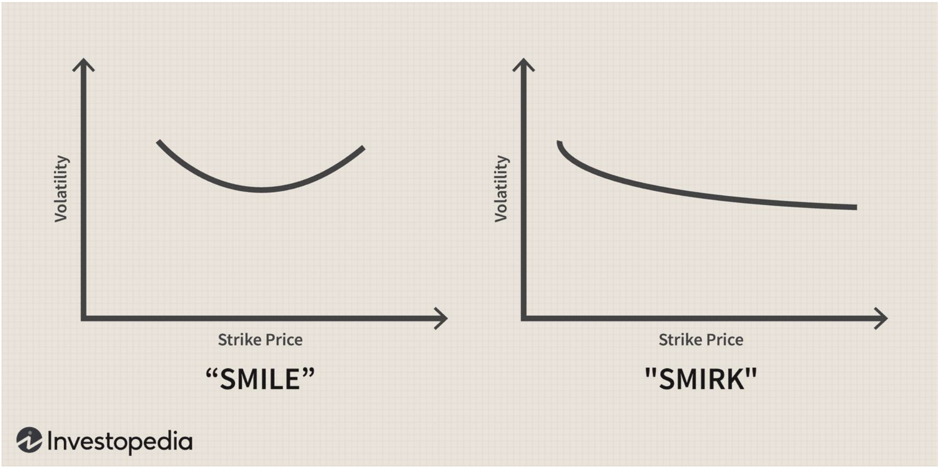
Problems With the Black-Scholes Model

- One of the most popular but outdated models which uses partial differential equations to price options
- Uses five input variables- **the strike price of an option, the current stock price, the time to expiration, the risk-free rate, and the volatility.**

Limitations of Black Scholes Model:

1. **Assumes Constant Volatility**- Assumes volatility remains constant over the option's life. However, volatility fluctuates with the level of supply and demand.
2. **Other Assumptions**- No transaction cost or taxes, and risk-free interest rate is constant for all maturities, and all these assumptions can lead to prices that deviate from actual results.
3. Only used to price European options, and does not take **American options** into account where options can be **exercised at any point before expiration**





How a GARCH Model Addresses These Issues

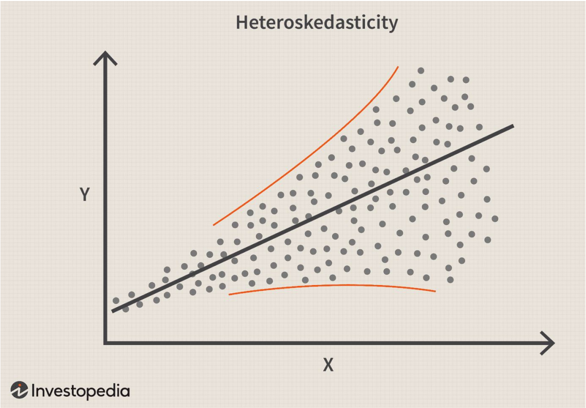
The **Generalized Autoregressive Conditional Heteroskedasticity (GARCH)** model is used for modeling volatility in financial time series.

GARCH Model forecasts the volatility of financial returns over time. It explicitly **allows volatility** to vary over time and capture volatility clustering, where periods of high volatility tend to be followed by additional periods of high volatility. The model also includes an autoregressive component for **capturing past volatility and a conditional heteroskedasticity component** that models the variability of the series conditional on past observations.

In practice, the estimated volatility from a GARCH model can be used as an input in option pricing models to account for changing volatility conditions.

Explicit Issues Addressed:

- **Volatility Clustering:** Financial time series often exhibit periods of high volatility followed by periods of low volatility. GARCH models are **well-suited to capture this clustering of volatility**.
- **Time-Varying Volatility:** GARCH models allow for the estimation of time-varying volatility, **addressing the limitation of assuming constant volatility** in the Black-Scholes model.



Q&A